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HIGH-VELOCITY PENETRATION OF PLANE SHAPED-CHARGE JETS INTO NONLINEAR MEDIA

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This paper reports results of metallographic analysis of metal samples cut from targets penetrated by plane shaped-charge jets. It is shown that the plastic deformation due to penetration has a turbulent nature and, in some cases, it occurs in metals with fractal structure formed after passage of the shock wave running ahead of the jet. A penetration model is proposed that takes into account the nonlinear behavior of the target material and the fractality of its structure.

Key words: *high-velocity penetration, shaped-charge jets, fractal structures.*

The high-velocity penetration of plane shaped-charge jets at initial impact velocities $V_0 = 2.5$–3.5 km/sec is accompanied by formation of a shock wave, which first runs ahead of the jet and then breaks up into an elastic wave and a plastic wave. The motion of the shock wave results, in particular, in local overheating of the target material and variations in its crystal and grain structures. Therefore, on most of the jet motion path, the jet penetrates into the target material whose properties differ from the initial ones. Examination of a number of targets from 12Kh18N10T steel and KhN75VMYu alloy penetrated by plane shaped-charge jets at initial impact velocities of about 3.5 km/sec using optical, scanning, and transmission microscopy revealed special features of the metal structure which indicate that the fractal structure formed after passage of the shock wave running ahead of the jet.

Structural variations in the alloy are shown on a panorama of the plane of the sample parallel to the direction of jet motion (Fig. 1). The sample is cut from the target at half-depth of the V-shaped cavern. These variations differ in etching regions, whose dimensions and shape are also different. According to these differences, three zones can be distinguished. The first zone (the zone of the largest plastic strains) of length $\Delta r_1 = (0.8 \pm 0.1)$ mm apparently forms during motion of the shaped-charge jet and consists of two regions. In the first region adjoining the cavern edge, whose dimensions do not exceed three or four grain sizes of the starting material, mixing of the jet and target materials occurs together with fragmentation of the target material and formation of discontinuities in it. In the second region of the first zone, fragmentation is accompanied by intense plastic deformation. The second zone with a dimension $\Delta r_2 = (1.1 \pm 0.1)$ mm formed as a result of shock-wave passage; i.e., the jet penetrate into the material with exactly this structure. Finally, the third zone corresponds to the initial metal structure with grain sizes of (50 ± 25) μm.

The most interesting (from the standpoint of the structure of the material into which the jet penetrates) is the second zone, which is an aggregate of strip structures oriented equally at an angle of $\pm 45^\circ$ to the cavern edge and is, in essence, a group soliton consisting of 7–12 monochromatic bands. As is known, the occurrence of solitons (solitary standing waves) is explained by nonlinear behavior of a medium. In the case considered, the nonlinear medium has fractal properties, as can be seen on images of the grain boundaries and the strip regions formed by intersection of slip microbands inside grains (Fig. 2) with increase in the microscope magnification and resolution. In these regions, the average fractal dimension D determined by the method of [1] is 1.896.

In the case of a target from 12Kh18N10T steel, the dislocation–disclination structure of the sample in the zone of shock wave propagation (Fig. 3) has the property of geometrical self-similarity. The dimensions of subregions subjected to local reorientation are approximately equal to 0.01, 0.05, and 0.26 μm. The results of the divisions

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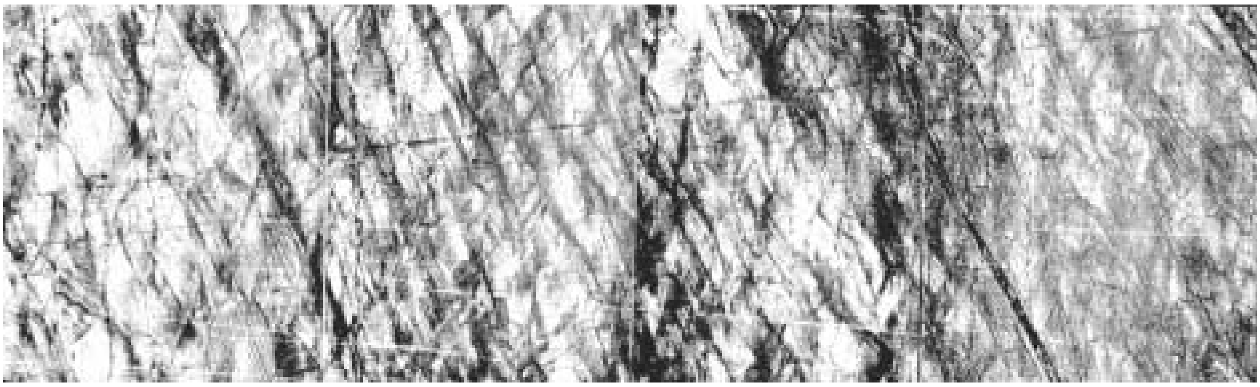

Fig. 1. Structure of KhN75VMYu alloy after penetration of a plane shaped-charge jet (×200).

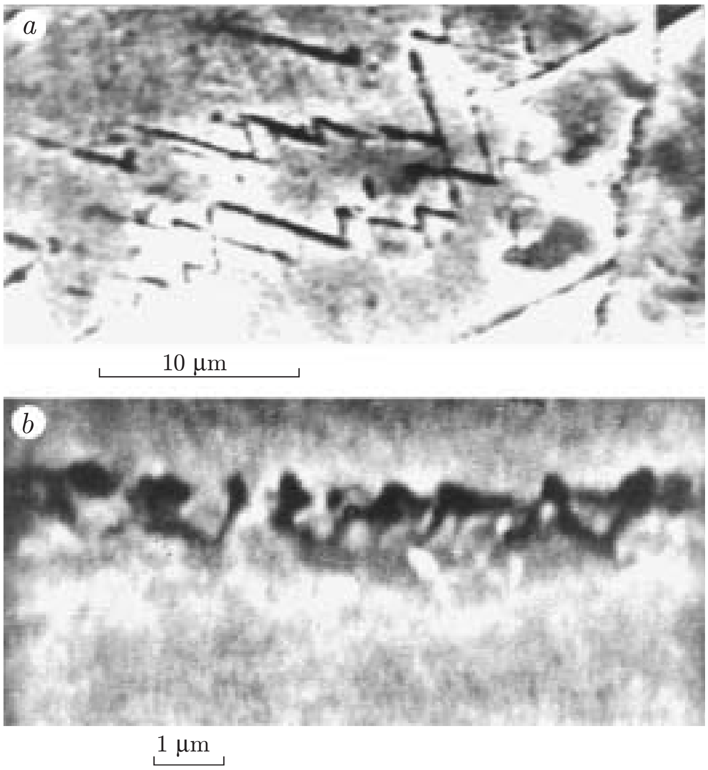

Fig. 2. Fractal grain boundary in KhN75VMYu alloy: (a) strip structures inside grains; (b) boundary of a strip at increased optical magnification.

0.01 : 0.05 and 0.05 : 0.26 are 0.2 and 192, respectively, and, as is known, the geometrical self-similarity of the objects is a classical feature of fractals.

The above data suggest that for some metals, the shock wave that arises from impact of a high-velocity jet on a target and runs ahead of the jet promotes the formation of fractal structures in the target material. As a result, the plane shaped-charge jet moves in a fractal medium with special nonlinear properties. Barakhtin et al. [2] established that the most intense deformation and fracture processes are localized within three or four grains located at the cavern edge. This is supported by the results of the present study. Barakhtin et al. [3] showed that plastic deformation is the motion of an aggregate of material microflows and mesoflows having different velocities. This conclusion is confirmed by the fact that the group soliton admits that on the phase plane there exist carrier paths of plastic modes in the form of a separatrix — a curve that separates random carriers according to velocities [4]. If the material microflows or mesoflows move in the same direction but at different velocities, a zone of turbulence (eddy flow), which also has fractal structure, forms on the boundary between these flows [4].

Fig. 3. Fine structure of 12Kh18N10T steel (×12,000).

In a rigorous formulation, the rather complex solution of the three-dimensional unsteady problem of determining the parameters of jet motion in a target becomes even more complicated in describing the penetration process in nonlinear fractal media because the classical models of continuum mechanics are unsuitable for describing their elastic and other properties. Thus, fractional derivatives and fractional integrals appear in the equations of motion and conservation [5], and the physical properties of fractal materials are determined by dependences of material density on its structure and dependences of elastic modulus on deformation scale [6].

To take into account the above-mentioned features of high-velocity penetration and to simplify the problem, we use the modified hydrodynamic model of penetration from [3]; the fractality is allowed for by effective parameters of material.

In the modified hydrodynamic penetration theory, the basic equation describing the stress balance on the jet–target interface is written as

$$0.5\rho_1(V-u)^2 + H_1 = 0.5\rho_2 u^2 + H_2, \tag{1}$$

where ρ_1 and ρ_2 are the densities of the jet and target materials, respectively, V is the jet velocity, u is the penetration velocity, H_1 is a parameter that describes the hardness of the jet material, and H_2 is the so-called resistance of the target material [7]. To solve Eq. (1) and the equations of the modified hydrodynamic penetration model, we need to know the values of the parameters H_1 and H_2. These characteristics were analyzed primarily for axisymmetric jets with velocities $V \approx 4$ km/sec and impactors (long rods and anvil blocks) with velocities 1.5–.5 km/sec, which are close to the velocities considered in the present paper. Tate recommended a value of H_1 equal to the Hugoniot elastic limit p_E of the impactor material and a value of H_2 which is 3.5 times larger than the Hugoniot elastic limit of the target material [8]. Chou and Flis [9] propose to use a value of H_1 equal to the dynamic yield limit and a value of H_2 equal to $2.5p_E$. In the domestic literature, the difference $H_2 - H_1$ for shaped-charge jets is understood mainly as the dynamic hardness of the target material [10] and it is assumed that the jet hardness can be ignored. Ulyakov [11] believes that H_1 and H_2 are close to the corresponding Hugoniot elastic limits, except in the case where the impactor and target materials are identical. In [7, 10], it is noted that from the viewpoint of physics, H_2 should be treated as dissipative pressure that describes energy losses due to elastoplastic flow, compressibility, target heating, etc. It should be noted that the values of the parameters H_1 and H_2 proposed above ignore the loading prehistory of the jet (impactor) and target materials (effect of shock-wave action), and, in essence, the difference $H_2 - H_1$ is a fitting parameter in these cases.

In determining jet strength parameters, Savenkov and Vasil'ev [12] obtained values $H_1 = 400$–700 MPa (mean value $H_1 = 470$ MPa) for M-2 copper, a mean value $H_1 = 520$ MPa for St. 3 steel, and $H_1 = 210$ MPa for A-6 aluminum. The values of H_1 obtained are close to the corresponding Hugoniot elastic limits and are not negligible.

In determining H_2, we shall take into account all above-mentioned physical features of jet penetration into metallic targets, i.e., we shall treat this parameter as the resistance of the target metal to jet penetration due to viscous friction between mesoflows and inside them, turbulent motion of particles of the medium in the intermediate sublayer, and rotational motion of grains and their fragments. The rotation of elements of the medium is suggested by diffusion of the target metal texture in the zone of the largest plastic strains (first zone) [3, 13]. It should be noted that the rotation of elements of the medium, as well as their motion (slip), is primarily an accommodation process contributing to the conservation of the continuity of the material.

In view of the aforesaid, the expression for H_2 is written as

$$H_2 = S_1 + S_2 + S_3 + S_4 + S_0, \tag{2}$$

were S_1 is the stress of friction between mesoflows, S_2 is the stress due to viscous friction of elements of the medium inside a mesoflow, S_3 is the turbulent stress, S_4 the rotational stress, and S_0 is the initial (taking into account shock-wave passage) strength of the target material.

The friction stress between mesoflows S_1 is given by the relation

$$S_1 = \mu_1 \dot{\varepsilon}_1, \tag{3}$$

where μ_1 is the dynamic viscosity of the material between mesoflows and $\dot{\varepsilon}_1$ is the strain rate between mesoflows.

The coefficient μ_1 is obtained from the following expression [14]:

$$\mu_1 = \rho_2 \, \Delta u \, \Delta h.$$

Here Δu is the width of the particle velocity distribution in a mesoflow (fluctuating velocity in the nomenclature of turbulent hydrodynamics, i.e., the change in velocity compared to the mean value) and Δh is the mesoflow width. The strain rate between mesoflows $\dot{\varepsilon}_1 = \Delta u/\Delta h$. Substituting the expressions for μ_1 and $\dot{\varepsilon}_1$ into (3), we obtain

$$S_1 = \rho_2 (\Delta u)^2. \tag{4}$$

The stress S_2 inside a mesoflow is due to the viscous drag of elementary plastic-strain carriers (dislocations). Similarly to S_1, it can be defined as

$$S_2 = \mu_2 \dot{\varepsilon}_2, \tag{5}$$

where μ_2 is the dynamic viscosity of the material inside a mesoflow and $\dot{\varepsilon}_2$ is the strain rate inside a mesoflow. The coefficient μ_2 is determined from the relation [14]

$$\mu_2 = \alpha B / (\boldsymbol{b}^2 N_m),$$

and the strain rate $\dot{\varepsilon}_2$ from the expression

$$\dot{\varepsilon}_2 = u/h \approx u/h_1.$$

Here $\alpha < 1$ is a coefficient, B is the viscous drag of dislocations, $\boldsymbol{b}$ is the Burgers vector, N_m is the density of mobile dislocations, h is the width of the zone of plastic strains, and h_1 is the width of the plane shaped-charge jet. Substituting the expressions for μ_2 and $\dot{\varepsilon}_2$ into (5), we obtain

$$S_2 = \alpha B u / (h_1 \boldsymbol{b}^2 N_m). \tag{6}$$

The turbulent (S_3) and rotational (S_4) stresses are the most difficult to determine. Formally, we can write

$$S_3 = \mu_3 \dot{\varepsilon}_3, \tag{7}$$

where μ_3 is the turbulent viscosity and $\dot{\varepsilon}_3$ is the strain rate in the turbulent zone. The coefficient μ_3 is obtained from the dependence defined in [15] as

$$\mu_3 = \lambda \rho_2 u d$$

(λ is an empirical fitting coefficient). The dimension of the turbulent zone d is determined from the relation

$$d = 0.5\beta f(\rho_{21}/\rho_{22})|u_1 - u_2|t_3, \tag{8}$$

where β is an empirical constant, $f(\rho_{21}/\rho_{22})$ is a dimensionless function, which generally depends on the density difference outside and inside the turbulent zone and is normalized by the condition $f(1) = 1$, and t_3 is the time of interaction of mesoflows. The velocity difference of two neighboring mesoflows $|u_1 - u_2|$ can be determined using laser interferometry data on failure of pulsations on interferograms [16]: $|u_1 - u_2| \approx \delta u$. Let $\dot{\varepsilon}_3 \approx 1/t_3$; then, with allowance for (8), relation (7) becomes

$$S_3 = \gamma \rho_2 u\, \delta u, \tag{9}$$

where $\gamma = 0.5\lambda\beta$.

It should be noted that Δu and δu are functions of the penetration velocity u; in this case, $\Delta u = f(0) = 0$ and $\delta u = f(0) = 0$.

The stress S_4 due to the rotation of grains and their fragments can formally be defined similarly to (7):

$$S_4 = \mu_4 \dot{\varepsilon}_4.$$

In this case, the dynamic viscosity μ_4 is defined by the relation proposed in [14]:

$$\mu_4 = [(0.1\rho_2 E)^{0.5} \omega d_0 - \sigma_0]/\dot{\varepsilon}_4,$$

Here E is Young's modulus, ω is the rotation angle of a grain (fragment) with mean size d_0, and σ_0 is the yield point. As a result, we obtain

$$S_4 = (0.1\rho_2 E)^{0.5} \omega d_0 - \sigma_0. \tag{10}$$

The angular velocity of rotation of a grain (fragment) can be found under the assumption that the angular momentum arises from scatter in the translational velocities of the grains (fragments). Then, provided that the velocity scatter is equal to $\delta u/2$, we have

$$\omega \approx \delta u/(0.4r), \tag{11}$$

where r is the mean radius of a grain (fragment) of spherical shape (the spherical shape explains the use of a value of 0.4 for the coefficient).

Substituting (11) into (10), we finally obtain

$$S_4 \approx 0.8(\rho_2 E)^{0.5}\, \delta u - \sigma_0. \tag{12}$$

As S_0, we use the stress value for which the target material with the structure formed after shock wave passage enters a plastic state (Hugoniot's elastic limit):

$$S_0 = \frac{1-\nu}{1-2\nu}\, \sigma_0 \tag{13}$$

(ν is Poisson's ratio).

The coefficient ν is determined from the relation between Young's modulus E and the bulk compression modulus K:

$$\nu = 0.5 - E/(6K). \tag{14}$$

For fractal materials, the elastic constants depend on the strain scale [6] and these dependences are written as

$$E = E_0 \lambda_1^{-m}, \qquad K = K_0 \lambda_1^{-m_1}. \tag{15}$$

Here λ_1 is the scale factor and m and m_1 are the geometrical factors of elasticity. [The scale dependence of Young's modulus should also be included in relation (12).]

TABLE 1

Target material	$\rho_2 \cdot 10^{-3}$, kg/m^3	E_0, GPa	K_0, GPa	σ_i, MPa	K_*, MPa $\cdot$ mm$^{0.5}$	D	d_0, μm	$\varphi \cdot 10^{-2}$
KhN75VMYu	8.10	184	112	390	110	1.896	50	8
12Kh18N10T	7.71	146	124	255	25	1.810	75	2

Notes. 1) For both materials, $m_1 = 1.9318$, $m = 0.9675$, and $\lambda_1 = 0.2$; 2) The Young's modulus E_0 and the bulk compression modulus K_0 correspond to a temperature $T = 400°$C.

Substituting (15) into (14), for fractal materials we obtain

$$\nu_{\rm f} = 0.5 - E_0\lambda_1^{m_1-m}/(6K_0). \tag{16}$$

Because $m_1 > m$ [6] from general considerations and $\lambda_1 < 1$ (though this is not obvious), it follows that for fractal materials, $\nu_{\rm f} > \nu$, and according to (13), the value of S_0 for them is larger than those for conventional materials.

The yield point σ_0 also depends on the shock wave generated after shock wave passage through the structure of the material, and its determination is hampered because of the small dimensions of the witness zone of the material. In this paper, we only estimate the effect of the fractality (nonlinearity) of the medium on the jet penetration depth. For this, we consider the well-known Hall–Petch relation (or a similar relation for fragmented media), which links the yield limit of a polycrystal and the grain (fragment) size:

$$\sigma_0 = \sigma_i + K_* d_0^{-0.5} \tag{17}$$

(σ_i and K_* are constants).

According to the Lie model [17],

$$K_* = \alpha_1 N^{0.5}. \tag{18}$$

Here α_1 is the proportionality constant and N is the number of protrusions on the fractal boundary of the grain:

$$N = \alpha_2 L^D, \tag{19}$$

α_2 is the fitting coefficient and L is the mean length of the grain boundary.

Substituting (16)–(19) into (13) and taking into account that $L \approx \pi d_0$, we finally obtain

$$S_0 = \frac{0.5 + a\lambda_1^{m_1-m}}{2a\lambda_1^{m_1-m}}\,(\sigma_i + \varphi\pi^{0.5D} d_0^{D-0.5}) \tag{20}$$

[$a = E_0/(6K_0)$ and $\varphi = \alpha_1\alpha_2^{0.5}$]. Then, with allowance for (4), (6), (9), (12), and (20), relation (2), defining the resistance H_2, becomes

$$H_2 = \rho_2(\Delta u)^2 + \frac{\alpha B}{h_1 N_m b^2}\,u + \rho_2\gamma u\,\delta u + 0.8(\rho_2 E)^{0.5}\delta u + \frac{0.5 - a\lambda_1^{m_1-m}}{2a\lambda_1^{m_1-m}}\,(\sigma_i + \varphi\pi^{0.5D} d_0^{0.5(D-1)}).$$

The process of high-velocity penetration has several stages [18]. At the first, unsteady, stage, jet penetration proceeds without a noticeable plastic strain in the target [2]. Therefore, in the relation for H_2, we can ignore the first, third, and fourth terms and set $\lambda_1 = 1$ and $D = 0$ in the fifth term. At the third, also unsteady, stage $\Delta u \approx 0$ and $\delta u \approx 0$ during jet deceleration, and, thus, the first, third, and fourth terms are equal to zero.

Using the expression for H_2, from Eq. (1) we find the critical jet velocity $V_{\rm cr}$ at which the jet penetration into the target stops ($u = 0$). As a result, we obtain

$$V_{\rm cr} = \sqrt{\frac{2}{\rho_1}\Big(\frac{0.5 - a\lambda_1^{m_1-m}}{2a\lambda_1^{m_1-m}}\,(\sigma_i + \varphi\pi^{0.5D} d_0^{0.5(D-1)}) - H_1\Big)}.$$

In calculating $V_{\rm cr}$ and H_2, we also need to allow for the temperature dependence of the elastic moduli E_0 and K_0 because for most metals, the values of these characteristics considerably decrease even at a temperature above 350°C and the target heating due to shock-wave passage at the second and third stages of jet penetration is not less than 400°C.

From evaluations for targets from KhN75VMYu alloy and 12Kh18N10T steel (parameter values are given in Table 1) and a copper jet ($\rho_1 = 8.9 \cdot 10^3$ kg/m^3 and $H_1 = 470$ MPa), it follows that $V_{\rm cr} = 875$ m/sec for KhN75VMYu alloy and $V_{\rm cr} = 675$ m/sec for 12Kh18N10T steel.

The obtained values of $V_{\rm cr}$ agree with available data, are in the real range of velocities, and are close to the values of $V_{\rm cr}$ found in [3]. Generally, in spite of a large number of coefficients to be determined, this indicates that the proposed model is suitable for describing the process of high-velocity penetration of plane shaped-charge jets.

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